Since when \( f(x) \) is divided by \( 2x + 3 \), the remainder is 21, we have
\[
f\left(\frac{-3}{2}\right) = 21
\]
i.e.
\[
2 \left(\frac{-3}{2}\right)^2 + q \left(\frac{-3}{2}\right) + p = 21
\]
\[
\frac{-3p}{2} + q = \frac{51}{2}
\]
\[
(i)
\]
\[
(ii)
\]
Substitute \( p = -13 \) into (i),
\[
\frac{-13}{2} + q = \frac{1}{2}
\]
\[
q = \frac{1}{2}
\]

(b) From (a), \( f(x) = 2x^3 + x^2 - 13x + 6 \)
Using long division, we get
\[
f(x) = (2x - 1)(x^2 + x - 6)
\]
\[
= (2x - 1)(x + 3)(x - 2)
\]

28. (a) \( x^2 + x - 6 = (x - 2)(x + 3) \)

(b) Since \( x - 2 \) is a factor of \( x^2 + x - 6 \) and \( x^2 + x - 6 \) is a factor of \( f(x) \), \( x - 2 \) is a factor of \( f(x) \).
\[
\therefore f(2) = 0
\]

Since \( x + 3 \) is a factor of \( x^2 + x - 6 \) and \( x^2 + x - 6 \) is a factor of \( f(x) \), \( x + 3 \) is a factor of \( f(x) \).
\[
\therefore f(-3) = 0
\]

(c) \( f(x) = ax^3 + x^2 - 27x + b \)
From (b),
\[
f(2) = 0
\]
i.e.
\[
a(2)^3 + 2^2 - 27(2) + b = 0
\]
\[
8a + b = 50 \quad (i)
\]

From (b),
\[
f(-3) = 0
\]
i.e.
\[
a(-3)^3 + (-3)^2 - 27(-3) + b = 0
\]
\[
-27a + b = -90 \quad (ii)
\]

\[
(i) - (ii): \quad 35a = 140
\]
\[
a = 4
\]
Substitute \( a = 4 \) into (i),
\[
8(4) + b = 50
\]
\[
b = 18
\]
1. (a) \[ \frac{x^2 - 6x + 10}{x + 2} \]
\[ \frac{x^3 - 4x^2}{x^2 + 2x} \]
\[ - 6x^2 - 2x \]
\[ - 6x^2 - 12x \]
\[ 10x + 1 \]
\[ 10x + 20 \]
\[ - 19 \]
\[ \therefore \text{Quotient} = \frac{x^2}{x - 6x + 10}, \]
\[ \text{remainder} = -19 \]

(b) \[ \frac{5x^3 + 6x + 11}{x - 3} \]
\[ \frac{5x^3 - 9x^2 - 7x - 2}{5x^3 - 15x^2} \]
\[ 6x^2 - 7x \]
\[ 6x^2 - 18x \]
\[ 11x - 2 \]
\[ 11x - 33 \]
\[ 31 \]
\[ \therefore \text{Quotient} = \frac{5x^2}{5x^3 + 6x + 11}, \]
\[ \text{remainder} = 31 \]

(c) \[ \frac{3x^2 - 2x + 1}{2x + 1} \]
\[ \frac{6x^3 - x^2 + 0x - 5}{6x^2 + 3x^2} \]
\[ - 4x^2 + 0x \]
\[ - 4x^2 - 2x \]
\[ 2x - 5 \]
\[ 2x + 1 \]
\[ - 6 \]
\[ \therefore \text{Quotient} = \frac{3x^2}{3x^3 - 2x + 1}, \]
\[ \text{remainder} = -6 \]

2. \[ \frac{2x^2 + x - 3}{x + 5} \]
\[ \frac{2x^3 + 11x^2 + 2x - 8}{2x^3 + 10x^2} \]
\[ x^2 + 2x \]
\[ x^2 + 5x \]
\[ - 3x - 8 \]
\[ - 3x - 15 \]
\[ 7 \]
\[ \therefore \text{Each child will get} \ (2x^2 + x - 3) \ \text{pieces of candy, 7 pieces of candy are left.} \]

3. (a) \[ 6x^2 - 13x - 21 = (3x + 4) \times \text{divisor} + 7 \]
\[ 6x^2 - 13x - 21 - 7 = (3x + 4) \times \text{divisor} \]
\[ 6x^2 - 13x - 28 = (3x + 4) \times \text{divisor} \]
\[ \text{divisor} = (6x^2 - 13x - 28) \div (3x + 4) \]
\[ = 2x - 7 \]
\[ \therefore \text{The required divisor is} \ 2x - 7. \]

(b) \[ 4x^3 + 11x^2 - 5x + 2 \]
\[ = (4x - 1) \times \text{divisor} + (2x + 1) \]
\[ 4x^3 + 11x^2 - 5x + 2 - (2x + 1) \]
\[ = (4x - 1) \times \text{divisor} \]
\[ 4x^3 + 11x^2 - 7x + 1 = (4x - 1) \times \text{divisor} \]
\[ \text{divisor} = (4x^3 + 11x^2 - 7x + 1) \div (4x - 1) \]
\[ = x^2 + 3x - 1 \]
\[ \therefore \text{The required divisor is} \ x^2 + 3x - 1. \]

4. (a) Dividend = \[ (3x + 4)(x + 2) + (-10) \]
\[ = 3x^2 + 6x + 4x + 8 - 10 \]
\[ = 3x^2 + 10x - 2 \]
\[ \therefore \text{The required dividend is} \]
\[ \frac{x^2 + 10x - 2}{3x^2 + 10x - 2} \]

(b) Dividend = \[ (2x + 5)(2x^2 - 1) + (-x + 6) \]
\[ = 4x^3 - 2x + 10x^2 - 5 - x + 6 \]
\[ = 4x^3 + 10x^2 - 3x + 1 \]
\[ \therefore \text{The required dividend is} \]
\[ \frac{x^2 + 10x - 2}{4x^3 + 10x^2 - 3x + 1} \]
5. (a) Let \( f(x) = 4x^2 - 2x - 7 \).

\[
\text{Remainder} = f\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^2 - 2\left(-\frac{3}{2}\right) - 7 = 5.
\]

(b) Let \( f(x) = x^3 - 3x^2 + 4 + 5x \).

\[
\text{Remainder} = f(0) = 0^3 - 3(0)^2 + 4 + 5(0) = 4.
\]

6. Let \( f(x) = x^2 - 6kx - 3 \).

\[
\therefore \text{Remainder} = 9k + 7
\]

\[
\therefore f(-2) = 9k + 7
\]

i.e. \((-2)^2 - 6k(-2) - 3 = 9k + 7
= 12k + 1 = 9k + 7
= 3k = 6
= k = 2
\]

7. Let \( f(x) = x^3 + x^2 - 3x - 20 \).

\[
\therefore \text{Remainder} = a^3 - 2
\]

\[
\therefore f(a) = a^3 - 2
\]

i.e. \(a^3 + a^2 - 3a - 20 = a^3 - 2
= a^2 - 3a - 18 = 0
= (a - 6)(a + 3) = 0
= a = 6 \text{ or } -3
\]

8. \( f(x) = x^3 - 3x^2 - 4x + 12 \)

(a) \( f(3) = 3^3 - 3(3)^2 - 4(3) + 12 = 0 \)

(b) \( \therefore f(3) = 0 \)

\( \therefore x - 3 \) is a factor of \( f(x) \).

Using long division, we get

\[
\frac{x^3 + x^2 + 2x}{x^3 + 3x^2 - 7x + 8}
\]

\[
\frac{-4x^2 - 9x + 8}{x^3 - 4x^2 - 8}
\]

\[
-5x + 16
\]

\( \therefore \text{Quotient} = x - 4 \),

\( \text{remainder} = -5x + 16 \)

9. Let \( f(x) = x^3 + kx^2 - 3x + k \).

Since \( x + 1 \) is a factor of \( f(x) \), we have

\[
f(-1) = 0
\]

i.e. \((-1)^3 + k(-1)^2 - 3(-1) + k = 0
= 2k + 2 = 2
= k = -1
\]

10. (a) Let \( f(x) = x^3 + 8x^2 - x - 8 \).

\[
\therefore f(1) = 1^3 + 8(1)^2 - 1 - 8 = 0
\]

\( \therefore x - 1 \) is a factor of \( f(x) \).

Using long division, we get

\[
f(x) = (x - 1)(x^2 + 9x + 8)
\]

\[
= (x - 1)(x + 1)(x + 8)
\]

(b) Let \( f(x) = x^3 - 5x^2 + 11x - 15 \).

\[
\therefore f(3) = 3^3 - 5(3)^2 + 11(3) - 15 = 0
\]

\( \therefore x - 3 \) is a factor of \( f(x) \).

Using long division, we get

\[
f(x) = (x - 3)(x^2 - 2x + 5)
\]

(c) Let \( f(x) = x^3 - 3x^2 - 10x + 24 \).

\[
\therefore f(2) = 2^3 - 3(2)^2 - 10(2) + 24 = 0
\]

\( \therefore x - 2 \) is a factor of \( f(x) \).

Using long division, we get

\[
f(x) = (x - 2)(x^2 - x - 12)
\]

\[
= (x - 2)(x + 3)(x - 4)
\]

11. (a) \( \frac{x - 4}{x^2 + x + 2} \)

\[
x^3 + x^2 - 7x + 8
\]

\[
x^3 + x^2 + 2x
\]

\[
-4x^2 - 9x + 8
\]

\[
-4x^2 - 4x - 8
\]

\[
-5x + 16
\]

\( \therefore \text{Quotient} = x - 4 \),

\( \text{remainder} = -5x + 16 \)

(b) \( \frac{2x + 4}{x^2 + 0x - 5} \)

\[
2x^3 + 4x^2 - x - 6
\]

\[
2x^3 + 0x^2 - 10x
\]

\[
4x^2 + 9x - 6
\]

\[
4x^2 + 0x - 20
\]

\[
9x + 14
\]

\( \therefore \text{Quotient} = 2x + 4 \),

\( \text{remainder} = 9x + 14 \)
12. (a) \( x^3 - 3x + 10 = (x + 1) \times \text{divisor} + 12 \)
\( x^3 - 3x + 10 - 12 = (x + 1) \times \text{divisor} \)
\( x^3 - 3x - 2 = (x + 1) \times \text{divisor} \)
\( \text{divisor} = \frac{x^3 - 3x - 2}{x + 1} \)
\( = x^2 - x - 2 \)
\[ \therefore \] The required divisor is \( x^2 - x - 2 \).

(b) \( x^4 + 6x^3 - x^2 - 4x + 11 \)
\( = (x^2 + 4x - 6) \times \text{divisor} + (20x - 7) \)
\( x^4 + 6x^3 - x^2 - 4x + 11 - (20x - 7) \)
\( = (x^2 + 4x - 6) \times \text{divisor} \)
\( x^4 + 6x^3 - x^2 - 24x + 18 \)
\( = (x^2 + 4x - 6) \times \text{divisor} \)
\( \text{divisor} = \frac{x^4 + 6x^3 - x^2 - 24x + 18}{x^2 + 4x - 6} \)
\( = x^2 + 2x - 3 \)
\[ \therefore \] The required divisor is \( x^2 + 2x - 3 \).

13. (a) Dividend = \( (2x + 3)(x^2 - x - 4) + (x + 8) \)
\( = 2x^3 - 2x^2 - 8x + 3x^2 - 3x - 12 + x + 8 \)
\( = 2x^3 + x^2 - 10x - 4 \)
\[ \therefore \] The required dividend is \( 2x^3 + x^2 - 10x - 4 \).

(b) Dividend = \( (x^2 + 2x + 1)(x^2 - 2x + 1) + (5x - 3) \)
\( = x^4 - 2x^3 + x^2 + 2x^3 - 4x^2 + 2x + x^2 - 2x + 1 + 5x - 3 \)
\( = x^4 - 2x^2 + 5x - 2 \)
\[ \therefore \] The required dividend is \( x^4 - 2x^2 + 5x - 2 \).

14. (a) \( f(x) = 2x^3 - 5x^2 - x + k \)
\[ \therefore f(1) = 0 \]
\( 2(1)^3 - 5(1)^2 - 1 + k = 0 \)
\( -4 + k = 0 \)
\( k = 4 \)

(b) From (a), \( f(x) = 2x^3 - 5x^2 - x + 4 \)
\( f\left(\frac{-1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 5\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 4 \)
\( = 3 \)
\[ \therefore \] When \( f(x) \) is divided by \( 2x + 1 \), the remainder is 3.

15. (a) \( f(x) = x^3 + mx - 4 \)
\( f(-2) = (-2)^3 + m(-2) - 4 \)
\( = -8m - 12 \)
\( f(1) = 1^3 + m(1) - 4 \)
\( = m - 3 \)
Since \( R_3 \) is 6 more than \( R_2 \), we have
\( f(-2) - f(1) = 6 \)
i.e. \( -8m - 12 - (m - 3) = 6 \)
\( -9m = 15 \)
\( m = -5 \)

(b) From (a), \( f(x) = x^3 - 5x - 4 \)
\( f(-3) = (-3)^3 - 5(-3) - 4 \)
\( = -16 \)
\[ \therefore \] When \( f(x) \) is divided by \( x + 3 \), the remainder is -16.

16. Let \( f(x) = x^3 + kx^2 + 2 \)
and \( g(x) = 4x^3 - 2x^2 + kx + 2 \).
\[ \therefore f(1) = g\left(\frac{1}{2}\right) \]
\( k^3 + k(1)^2 + 2 = 4\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) + 2 \)
\( 3 + k = \frac{k}{2} + 2 \)
\( k = -2 \)

18. Let \( f(x) = x^3 - 5x^2 + px + q \)

19. Let \( g(x) = 4x^3 - 2x^2 + px + q \)
Divide \( f(x) \) by \( g(x) \)
\[ \therefore f(x) = q(x - 1)^2 \]
\( = (x - 1)(x - 2) \)
\( = 6 \)
\( x^2 - 2x + 2 \)
17. Let \( f(x) = x^2 - 2x + 1 \) and \( g(x) = 3x^2 + x - 4 \).
\[
\begin{align*}
(\text{i}) & : f(-p) = g(-p) \\
(\text{ii}) & : (-p)^2 - 2(-p) + 1 = 3(-p)^2 + (-p) - 4 \\
& \quad p^2 + 2p + 1 = 3p^2 - p - 4 \\
& \quad 2p^2 - 3p - 5 = 0 \\
& \quad (2p - 5)(p + 1) = 0 \\
& \quad p = \frac{5}{2} \text{ or } -1
\end{align*}
\]
Substitute \( p = -6 \) into (i), \(-6 + q = -4 \)
\[q = 2\]

18. Let \( f(x) = x^2(ax + 1) + bx + 3 \).
\[
\begin{align*}
(\text{i}) & : f\left(\frac{1}{2}\right) = a \\
(\text{ii}) & : \left(\frac{1}{2}\right)^2 a + \left(\frac{1}{2}\right) + b = a \\
& \quad \frac{a}{4} + \frac{b}{2} + 3 = a \\
& \quad 7a - 4b - 26 = 0 \quad \text{(I)} \\
(\text{iii}) & : f(-2) = b \\
& \quad (-2)^2 a(-2) + b(-2) + 3 = b \\
& \quad -8a + 4 - 2b + 3 = b \\
& \quad 8a + 3b - 7 = 0 \quad \text{(II)}
\end{align*}
\]
From (I), \( b = \frac{1}{4}(7a - 26) \) \quad \text{(III)}
Substitute (III) into (II),
\[
\begin{align*}
8a + 3\left[\frac{1}{4}(7a - 26)\right] - 7 &= 0 \\
8a + \frac{21a}{4} - \frac{39}{2} - 7 &= 0 \\
\frac{53a}{4} &= \frac{53}{2} \\
a &= 2
\end{align*}
\]
Substitute \( a = 2 \) into (III), \( b = \frac{1}{4}(7(2) - 26) \)
\[b = -3\]

19. Let \( g(x) \) be the quotient obtained when \( f(x) \) is divided by \((x - 1)(3x + 2)\), then
\[
\begin{align*}
(\text{i}) & : g(1) = f(1) = -4 \\
(\text{ii}) & : g\left(\frac{2}{3}\right) = f\left(\frac{2}{3}\right) \\
& \quad \left(\frac{2}{3}\right)^2 - 2\left(\frac{2}{3}\right) + 1 [3\left(\frac{2}{3}\right) + 2] + p\left(\frac{2}{3}\right) + q = 6 \\
& \quad \frac{2q}{3} = 6 \quad \text{(II)}
\end{align*}
\]
\[4 \text{ should be added to the polynomial} \]
\[
\frac{2x^3}{3} - \frac{3x^2}{5} + 5x - 8 + c \text{ so that the resulting polynomial is divisible by } x - 1.
\]

20. (a) Let \( f(x) = x^3 - px^2 + 2x + (p + 3) \).
Since \( x = -2 \) is a factor of \( f(x) \), we have
\[
\begin{align*}
(\text{i}) & : f(2) = 0 \\
& \quad 2^3 - p(2)^2 + 2(2) + (p + 3) = 0 \\
& \quad -3p + 15 = 0 \\
& \quad p = 5
\end{align*}
\]
(b) From (a), \( f(x) = x^3 - 5x^2 + 2x + 8 \)
Using long division, we get
\[
\begin{align*}
(\text{i}) & : f(x) = (x - 2)(x^2 - 3x - 4) \\
(\text{II}) & : f(x) = (x - 2)(x + 1)(x - 4)
\end{align*}
\]

21. (a) \( P(x) = 2x^3 + x^2 - 13x + a \)
Since \( P(x) \) is divisible by \( x + 3 \), we have
\[
\begin{align*}
(\text{i}) & : P(-3) = 0 \\
& \quad 2(-3)^3 + (-3)^2 - 13(-3) + a = 0 \\
& \quad -18 + 27 + 39 + a = 0 \\
& \quad a = 6
\end{align*}
\]
(b) From (a), \( P(x) = 2x^3 + x^2 - 13x + 6 \)
\[
\begin{align*}
(\text{i}) & : P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 13\left(\frac{1}{2}\right) + 6 \\
& \quad = 0 \\
(\text{II}) & : P(x) \text{ is also divisible by } 2x - 1.
\end{align*}
\]

22. Let \( c \) be the required number, then the polynomial \( 2x^3 - 3x^2 + 5x - 8 + c \) is divisible by \( x - 1 \).
Let \( f(x) = 2x^3 - 3x^2 + 5x - 8 + c \).
Since \( f(x) \) is divisible by \( x - 1 \), we have
\[
\begin{align*}
(\text{i}) & : f(1) = 0 \\
& \quad 2(1)^3 - 3(1)^2 + 5(1) - 8 + c = 0 \\
& \quad -4 + c = 0 \\
& \quad c = 4
\end{align*}
\]
\[4 \text{ should be added to the polynomial} \]
\[
2x^3 - 3x^2 + 5x - 8 \text{ so that the resulting polynomial is divisible by } x - 1.
\]
23. (a) \( f(x) = 10x^3 + mx^2 - x + 6 \)
Since \( f(x) \) is divisible by \( 5x - 3 \), we have
\[
f\left(\frac{3}{5}\right) = 0
\]
i.e.
\[
10\left(\frac{3}{5}\right)^3 + m\left(\frac{3}{5}\right)^2 - \frac{3}{5} + 6 = 0
\]
\[
\frac{9m}{25} + \frac{189}{25} = 0
\]
\[
m = -21
\]
\[\therefore f(x) = 10x^3 - 21x^2 - x + 6\]
Using long division, we get
\[
f(x) = (5x - 3)(2x^2 - 3x - 2)
\]
Therefore, when \( f(x) \) is divided by \( 5x - 3 \), the quotient is \( 2x^2 - 3x - 2 \).
i.e. \( 2x^2 - 3x - 2 = 2x^2 + nx - 2 \)
\[\therefore \text{The coefficients of } x \text{ are equal.}
\]
\[\therefore n = -3
\]
(b) From (a),
\[
f(x) = (5x - 3)(2x^2 - 3x - 2)
\]
\[
= (5x - 3)(2x + 1)(x - 2)
\]
24. \( f(x) = x^3 + 2x^2 + ax + b \)
Since \( f(x) \) is divisible by \( x - 2 \), we have
\[
f(2) = 0
\]
i.e.
\[
2^3 + 2(2)^2 + a(2) + b = 0
\]
\[
2a + b = -16 \quad \text{(i)}
\]
Since when \( f(x) \) is divided by \( x + 2 \), the remainder is 4, we have
\[
f(-2) = 4
\]
i.e.
\[
(-2)^3 + 2(-2)^2 + a(-2) + b = 4
\]
\[
-2a + b = 4 \quad \text{(ii)}
\]
\[\text{(i)} - \text{(ii)}: 4a = -20 \]
\[a = -5
\]
Substitute \( a = -5 \) into (i), \( 2(-5) + b = -16 \)
\[b = -6
\]
25. (a) \( f(x) = 3x^3 - 7x^2 + kx + 8 \)
Since \( x - 4 \) is a factor of \( f(x) \), we have
\[
f(4) = 0
\]
i.e.
\[
3(4)^3 - 7(4)^2 + k(4) + 8 = 0
\]
\[
88 + 4k = 0
\]
\[k = -22
\]
(b) From (a), \( f(x) = 3x^3 - 7x^2 - 22x + 8 \)
\[
f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - 7\left(\frac{1}{3}\right)^2 - 22\left(\frac{1}{3}\right) + 8
\]
\[= 0
\]
\[\therefore \text{When } f(x) \text{ is divided by } 3x - 1, \text{ the remainder is 0.}
\]
26. (a) Let \( f(x) = x^3 + 4x^2 - 4x - 21 \).
\[\therefore f(-3) = (-3)^3 + 4(-3)^2 - 4(-3) - 21 = 0
\]
\[\therefore x + 3 \text{ is a factor of } f(x).
\]
Using long division, we get
\[f(x) = (x + 3)(x^2 + x - 7)
\]
(b) Let \( f(x) = 3x^3 + 5x^2 - 26x + 8 \).
\[\therefore f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^2 - 26\left(\frac{1}{3}\right) + 8 = 0
\]
\[\therefore 3x - 1 \text{ is a factor of } f(x).
\]
Using long division, we get
\[f(x) = (3x - 1)(x^2 + 2x - 8)
\]
\[= (3x - 1)(x - 2)(x + 4)
\]
27. Let \( g(x) \) and \( px + q \) be the quotient and the remainder obtained respectively when \( f(x) \) is divided by \( (x - 5)(x + 2) \), then
\[
f(x) = g(x) \cdot (x - 5)(x + 2) + (px + q).
\]
\[\therefore f(5) = 9
\]
\[\therefore g(5) \cdot (5 - 5)(5 + 2) + p(5) + q = 9
\]
\[5p + q = 9 \quad \text{(i)}
\]
\[\therefore f(-2) = -5
\]
\[\therefore g(-2) \cdot (-2 - 5)(-2 + 2) + p(-2) + q = -5
\]
\[-2p + q = -5 \quad \text{(ii)}
\]
\[\text{(i)} - \text{(ii)}: 7p = 14
\]
\[p = 2
\]
Substitute \( p = 2 \) into (i), \( 5(2) + q = 9 \)
\[q = -1
\]
\[\therefore \text{When } f(x) \text{ is divided by } (x - 5)(x + 2), \text{ the remainder is } 2x - 1.
\]
28. (a) Let \( g(x) \) be the quotient obtained when \( f(x) \) is divided by \( (x + 1)(x - 3) \), then
\[
f(x) = g(x) \cdot (x + 1)(x - 3) + (2x - 4).
\]
\[
f(-1) = g(-1) \cdot (-1 + 1)(-1 - 3) + 2(-1) - 4 = -6
\]
\[
f(3) = g(3) \cdot (3 + 1)(3 - 3) + 2(3) - 4 = 2
\]
\[
(b) \ f(x) = 3x^3 + mx^2 - nx - 7
\]
From (a), \( f(-1) = -6 \)
\[
\text{i.e. } 3(-1)^3 + m(-1)^2 - m(-1) - 7 = -6
\]
\[
m + n = 4
\]
From (a), \( f(3) = 2 \)
\[
\text{i.e. } 3(3)^3 + m(3)^2 - n(3) - 7 = 2
\]
\[
9m - 3n = -72
\]
\[
3m - n = -24
\]
\[
\therefore \text{ The required equations are } m + n = 4
\]
\[
\text{and } 3m - n = -24.
\]
(c) From (b), \( m + n = 4 \)
\[
.............. (i)
\]
\[
3m - n = -24 \quad .............. (ii)
\]
\[
(l) + (ii) \quad 4m = -20
\]
\[
m = -5
\]
\[
\text{Substitute } m = -5 \text{ into (i), } -5 + n = 4 \quad n = 9
\]

HKCEE Questions

(Paper 1 Questions)

29. 5

(Paper 2 Questions)

30. E

31. E

32. A

33. C

Solutions to (Open-ended) Questions

(P. 173)

\[ \therefore \text{ The degree of the dividend is the highest.} \]
\[ \therefore \text{ Dividend } = 2x^3 - 3x^2 + x - 50 \]

The degrees of the 3 remaining polynomials are 2, 1 and 1. Since the degree of the divisor must be greater than the degree of the remainder,
\[ \text{divisor } = 2x^2 + 5x + 20 \]

We can use different methods to distinguish the quotient and the remainder.

Method 1: Use long division to calculate
\[ \frac{2x^3 - 3x^2 + x - 50}{2x^2 + 5x + 20} \]
\[ x - 4 \]
\[
\begin{align*}
2x^2 + 5x + 20 &) 2x^3 - 3x^2 + x - 50 \\
\quad 2x^3 + 5x^2 + 20x & \\
\hline
\quad -8x^2 - 19x - 50 & \\
\quad -8x^2 - 20x - 80 & \\
\hline
\quad x + 30 &
\end{align*}
\]
\[ \therefore \text{Quotient } = x - 4, \]
\[ \text{remainder } = x + 30 \]

Method 2: Let the quotient be \( x + a \) and the remainder be \( x + b \), where \( a \) and \( b \) are constants,
\[ \text{then } 2x^3 - 3x^2 + x - 50 = (x + a)(2x^2 + 5x + 20) + (x + b). \]

Comparing the constant terms on the two sides of the above identity, we have
\[ -50 = 20a + b \quad ................. (\star) \]

Since the quotient or the remainder can only be one of the polynomials \( x - 4 \) and \( x + 30 \),
\[ \text{i.e. } a = -4, b = 30 \text{ or } a = 30, b = -4 \]

After checking, we know that only \( a = -4, b = 30 \) satisfy \( (\star) \).
\[ \therefore \text{Quotient } = x - 4, \]
\[ \text{remainder } = x + 30 \]