1. \( C \)

Drop a perpendicular from \( V \) to \( AB \), we have \( VM \) as an attitude of \( \triangle VAB \).
Let \( h \) cm be the length of \( VM \).
\[
AM = \frac{1}{2} \times 10 \text{ cm} \\
= 5 \text{ cm}
\]
Consider \( \triangle VAM \).
\[
h^2 + 5^2 = 10^2 \quad \text{(Pyth. theorem)} \\
h^2 = 100 - 25 \\
= 75 \\
h = \sqrt{75} \\
= 5\sqrt{3}
\]
Surface area of \( VABC \)
\[
= \left( \frac{1}{2} \times 10 \times 5\sqrt{3} \right) \times 4 \text{ cm}^2 \\
= 100\sqrt{3} \text{ cm}^2
\]

2. \( D \)

Let \( r \) and \( R \) be the original and new radii of the balloon respectively.
Original surface area : new surface area = \( 1 : (1 + 44\%) = 1 : 1.44 \)
\[
r^2 : R^2 = 1 : 1.44 \\
r : R = \sqrt{1} : \sqrt{1.44} = 1 : 1.2 \\
r^3 : R^3 = 1^3 : 1.2^3 = 1 : 1.728
\]
Percentage increase in volume
\[
= \frac{1.728 - 1}{1} \times 100\% \\
= 72.8\%
\]
3. A

$A, (A + B)$ and $(A + B + C)$ are similar cones of base radii 1 cm, 3 cm and 5 cm respectively.

Volume of $A : \text{volume of } (A + B) : \text{volume of } (A + B + C)$

$= 1^3 : 3^3 : 5^3$

$= 1 : 27 : 125$

Volume of $A : \text{volume of } B : \text{volume of } C$

$= 1 : (27 - 1) : (125 - 27)$

$= 1 : 26 : 98$

Volume of $B : \text{volume of } C$

$= 26 : 98$

$= 13 : 49$

4. C

Let $h$ cm be the height of the cone.

Volume of the cone

$= \frac{1}{3} \times \pi \times 8^2 \times h \text{ cm}^3$

$= \frac{64\pi h}{3} \text{ cm}^3$

Volume of water in the vessel with a depth of 3 cm

$= \pi \times \left(\frac{24}{2}\right)^2 \times 3 \text{ cm}^3$

$= 432\pi \text{ cm}^3$

$\frac{64\pi h}{3} = 432\pi$

$h = 432\pi \times \frac{3}{64\pi}$

$= 20.25$

$\therefore$ The height of the cone is 20.25 cm.

5. B

Let $r$ cm be the radius of the hemisphere.

Volume of the solid sphere

$= \frac{4}{3} \pi \times 9^3 \text{ cm}^3$
Volume of the solid hemisphere

\[
\frac{1}{2} \times \frac{4}{3} \pi \times r^3 \text{ cm}^3
\]

\[
\frac{4}{3} \pi \times 9^3 = \frac{1}{2} \times \frac{4}{3} \pi \times r^3
\]

\[
r^2 = 2 \times 9^3
\]

\[
r = 1458
\]

\[
r = 11.3 \text{ (cor. to 3 sig. fig.)}
\]

\[
\therefore \text{ The radius of the hemisphere is } 11.3 \text{ cm.}
\]

6. A
Height of the cone: \(3r - r = 2r\)

Let \(c\) be the slant height of the cone.

\[
c^2 = (2r)^2 + r^2 \quad \text{(Pyth. theorem)}
\]

\[
c = \sqrt{5}r
\]

Surface area of the solid

\[
= \pi r (\sqrt{5}r) + \frac{1}{2} \times 4\pi r^2
\]

\[
= (2 + \sqrt{5})\pi r^2
\]

7. C
For I and III, the formulae are products of two lengths, hence they are formulae for area.
For II, the formula is a product of three lengths, hence it is a formula for volume.

8. D
In \(\triangle ABC\),

\[
\sin 30^\circ = \frac{a}{BC}
\]

\[
BC = \frac{a}{\sin 30^\circ} = 2a
\]
In \( \triangle BDC \),
\[
\tan \theta = \frac{2a}{BD}
\]
\[
BD = \frac{2a}{\tan \theta}
\]

9. C
\[
(cos \theta + sin \theta)^2 - tan \theta \cos^2 \theta
\]
\[
= (cos^2 \theta + 2 \cos \theta \sin \theta + sin^2 \theta) - \frac{\sin \theta}{\cos \theta} \cos^2 \theta
\]
\[
= (1 + 2 \cos \theta \sin \theta) - \sin \theta \cos \theta
\]
\[
= 1 + \sin \theta \cos \theta
\]

10. B
For I,
L.H.S. = \( \sin^2(90^\circ - \theta) + \cos^2 \theta \)
\[
= \cos^2 \theta + \cos^2 \theta
\]
\[
= 2 \cos^2 \theta
\]
\[
\neq R.H.S.
\]
\[\therefore\] I is not an identity.

For II,
L.H.S. = \( \tan(90^\circ - \theta) \sin \theta \)
\[
= \frac{1}{\tan \theta} \sin \theta
\]
\[
= \frac{\cos \theta}{\sin \theta} \sin \theta
\]
\[
= \cos \theta
\]
\[
= R.H.S.
\]
\[\therefore\] II is an identity.

For III,
L.H.S. = \( \frac{1 - \cos^2 \theta}{\cos(90^\circ - \theta)} \)
\[
= \frac{\sin^2 \theta}{\sin \theta}
\]
\[
= \sin \theta
\]
\[
\neq R.H.S.
\]
\[\therefore\] III is not an identity.
11. \[ A \]

\[ \cos^2(90^\circ - \theta) \tan(90^\circ - \theta) \]

\[ = \sin^2 \theta \frac{1}{\tan \theta} \]

\[ = \sin^2 \theta \frac{\cos \theta}{\sin \theta} \]

\[ = \sin \theta \cos \theta \]

\[ = ab \]

12. \[ C \]

\[ (\tan 60^\circ - \sin 60^\circ)(\cos 45^\circ + \sin 45^\circ) \]

\[ = \left( \sqrt{3} - \frac{\sqrt{3}}{2} \right) \frac{\sqrt{2} + \sqrt{2}}{2} \]

\[ = \frac{\sqrt{3}}{2} \times \sqrt{2} \]

\[ = \frac{\sqrt{6}}{2} \]

13. \[ D \]

\[ \sin \theta \tan \theta = \sin \theta \frac{\sin \theta}{\cos \theta} \]

\[ = \sin^2 \theta \frac{1}{\cos \theta} \]

\[ = 1 - \cos^2 \theta \frac{1}{\cos \theta} \]

\[ = 1 - \left( \frac{1}{2} \right)^2 \]

\[ = 1 - \frac{1}{4} \]

\[ = \frac{3}{2} \]
14. C

Let \( M \) be a point on \( CD \) such that \( BM \perp CD \).

\[
\tan 40^\circ = \frac{BM}{CM}
\]

\[
= \frac{BM}{y - x}
\]

\[
BM = (y - x) \tan 40^\circ
\]

Area of \( ABCD \)

\[
= \frac{1}{2} \times (x + y) \times BM
\]

\[
= \frac{1}{2} \times (x + y) \times (y - x) \tan 40^\circ
\]

\[
= \frac{(y^2 - x^2) \tan 40^\circ}{2}
\]

15. B

\( \angle RPS = 60^\circ + 45^\circ = 105^\circ \)

\( \angle TRP + \angle RPS = 180^\circ \)

\( \angle TRP = 180^\circ - 105^\circ \)

\( = 75^\circ \)

The compass bearing of \( P \) from \( R \) is \( S75^\circ W \).

16. C

Vertical distance of \( AB = (500 - 200) \text{ m} = 300 \text{ m} \)

Horizontal distance of \( AB = 2.5 \times 10000 \text{ cm} = 25000 \text{ cm} = 250 \text{ m} \)

\[ \therefore \text{ Gradient of } AB = \frac{300 \text{ m}}{250 \text{ m}} = \frac{6}{5} \]
17. B

Let \( \theta \) be the angle that the road makes with the horizontal.

\[
\tan \theta = \frac{1}{9}
\]

\( \theta \approx 6.3402^\circ \)

Let \( h \) km be the vertical distance the car rises.

\[
\sin \theta = \frac{h}{25}
\]

\( h \approx 25 \sin 6.3402^\circ \)

\( = 2.76 \) (cor. to 3 sig. fig.)

\( \therefore \) The car rises 2.76 km vertically.

18. B

Denoting Alice, Ben and Cindy by \( A \), \( B \) and \( C \) respectively, we have

\( \angle BAC = 180^\circ - 70^\circ - 35^\circ \)

\( = 75^\circ \)

\( \therefore \ BA = BC \)

\( \therefore \ \angle BCA = \angle BAC = 75^\circ \)

\( \therefore \ \angle DCA = \angle EAC = 35^\circ \)

\( \therefore \ \angle DCB = \angle BCA - \angle DCA \cdot \)

\( = 75^\circ - 35^\circ \)

\( = 40^\circ \)

\( \therefore \) The angle of depression of Ben from Cindy is \( 40^\circ \).

19. A

\( \angle LPM = 90^\circ - 55^\circ = 35^\circ \)

\( \angle LQM = 325^\circ - 270^\circ = 55^\circ \)

\( \angle PLQ = 180^\circ - \angle LPM - \angle LQM \)

\( = 180^\circ - 35^\circ - 55^\circ \)

\( = 90^\circ \)

Consider \( \triangle LPQ \).
\[
\cos \angle LPQ = \frac{LP}{PQ}
\]
\[LP = PQ \cos 35^\circ\]
Consider \(\triangle LPM\).
\[\sin \angle LPM = \frac{LM}{LP}\]
\[LM = LP \sin 35^\circ\]
\[= PQ \cos 35^\circ \sin 35^\circ\]
\[= PQ \sin (90^\circ - 35^\circ) \cos (90^\circ - 35^\circ)\]
\[= PQ \sin 55^\circ \cos 55^\circ\]

20. C
Consider \(\triangle PRS\).
\[\frac{PS}{PR} = \sin 60^\circ\]
\[PS = \frac{5\sqrt{3}}{2} \text{ cm}\]
Consider \(\triangle PQS\).
\[\frac{PS}{PQ} = \sin 30^\circ\]
\[PQ = 2PS\]
\[= 2 \times \frac{5\sqrt{3}}{2} \text{ cm}\]
\[= 5\sqrt{3} \text{ cm}\]
\[= 8.66 \text{ cm} \text{ (cor. to 3 sig. fig.)}\]

21. D
Let the coordinates of \(B\) be \((x, y)\).
\[3 = \frac{-2 + x}{2}\]
\[6 = -2 + x\]
\[x = 8\]
\[-1 = \frac{3 + y}{2}\]
\[-2 = 3 + y\]
\[y = -5\]
\[\therefore \quad B = (8, -5)\]
22.  D

\[ AR : RB = 3 : 1 \]

Let \((x, y)\) be the coordinates of \(R\).

\[ x = \frac{3(12) + 1(8)}{3 + 1} = 11 \]
\[ y = \frac{3(-6) + 1(14)}{3 + 1} = -1 \]

\[ \therefore \quad R = (11, -1) \]

23.  A

The diagonals \(AC\) and \(BD\) bisect each other.

\[ AM = MC \quad \text{and} \quad BM = MD \]

Let \((x, y)\) be the coordinates of \(M\).

\[ x = \frac{3 + 7}{2} = 5 \]
\[ y = \frac{6 + 1}{2} = \frac{7}{2} \]

\[ \therefore \quad M = (5, \frac{7}{2}) \]

Let \((x, y)\) be the coordinates of \(D\).

\[ 5 = \frac{1 + x}{2} \]
\[ 10 = 1 + x \]
\[ x = 9 \]
\[ 3.5 = \frac{-2 + y}{2} \]
\[ 7 = -2 + y \]
\[ y = 9 \]

\[ \therefore \quad D = (9, 9) \]

24.  C

Let \((x, 0)\) be the coordinates of \(P\) and \(AP : PB = 1 : r\).

\[ 0 = \frac{1(-5) + r(2)}{1 + r} \]

\[ 2r - 5 = 0 \]
\[ r = \frac{5}{2} \]

\[ AP : PB = 1 : \frac{5}{2} = 2 : 5 \]
25. B

\[ AC = 3AB \]
\[ BC = AC - AB \]
\[ = 3AB - AB \]
\[ = 2AB \]
\[ AB : BC = 1:2 \]

Let \((x, y)\) be the coordinates of \(C\).
\[ 3 = \frac{1(x) + 2(-1)}{1 + 2} \]
\[ 9 = x - 2 \]
\[ x = 11 \]
\[ 5 = \frac{1(y) + 2(1)}{1 + 2} \]
\[ 15 = y + 2 \]
\[ y = 13 \]

\[ \therefore C = (11, 13) \]

26. D

\[ -2 = \frac{1(b) + 2(2a)}{1 + 2} \]
\[ b + 4a = -6 \quad ...... (1) \]
\[ 1 = \frac{1(b + 3) + 2(a)}{1 + 2} \]
\[ 3 = 2a + b + 3 \]
\[ 2a + b = 0 \quad ...... (2) \]

\((1) - (2),\)

\[ 2a = -6 \]
\[ a = -3 \]

By substituting \(a = -3\) into (2), we have
\[ 2(-3) + b = 0 \]
\[ b = 6 \]

27. A

Five years ago, each student’s age was 5 years less than his/her present age.

Mean age = present age - 5
\[ = 13.5 - 5 \]
\[ = 8.5 \]
28. B
Mean number of children
\[ \frac{1 \times 20 + 2 \times 14 + 3 \times 6}{50} \]
\[ = \frac{50}{50} \]
\[ = 1.32 \]

29. C
From the graph, the mark corresponding to the cumulative frequency of 30 is 51.5.

30. A
Weighted mean mark of Jason
\[ \frac{83 \times 5 + 75 \times 3 + x \times 2}{5 + 3 + 2} \]
\[ = \frac{640 + 2x}{10} \]
\[ = 64 + \frac{x}{5} \]
\[ 64 + \frac{x}{5} = 80 \]
\[ \frac{x}{5} = 16 \]
\[ x = 80 \]

31. B
\[ \frac{3 + 5 + 5 + 9 + 9 + a}{6} = 6 \]
\[ 31 + a = 36 \]
\[ a = 5 \]
The numbers are 3, 5, 5, 5, 9 and 9.
\[ \therefore \text{ The mode is 5.} \]

32. B
Let \( x \) cm be the mean height of girls.
\[ 158 \times 0.55 + x \times 0.45 = 153.5 \]
\[ x = 148 \]
The mean height of girls is 148 cm.

33. C
34. D
Possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT
where H and T represent head and tail respectively.

\[ P(\text{at least 1 head}) = \frac{7}{8} \]

35. A
Possible outcomes: \( PX, PY, QX, QY, RX \) and \( RY \)

\[ P(\text{passes via road } Y) = P(PY, QY \text{ and } RY) = \frac{3}{6} = \frac{1}{2} \]

36. D
Expected value of the coin

\[ = $1 \times \frac{1}{5} + $5 \times \frac{2}{5} + $10 \times \frac{2}{5} \]

\[ = $6.2 \]

37. C
Possible outcomes: 10, 12, 13, 20, 21, 23, 30, 31, 32
Favourable outcomes: 10, 12, 20, 30, 32

\[ P(\text{even number}) = \frac{5}{9} \]

38. B

\[ P(\text{by MTR}) = \frac{360^\circ - 80^\circ - 130^\circ - 90^\circ}{360^\circ} = \frac{60^\circ}{360^\circ} = \frac{1}{6} \]
39. B

\[ x = 10000 - 1027 - 3153 - 2578 - 1226 - 703 \]

\[ = 1310 \]

Experimental probability of getting a '2'

\[ \frac{1310}{10000} \]

\[ = 0.131 \]

40. A

Denoting the two red, the black and the white balls by \( R_1, R_2, B \) and \( W \) respectively.

Possible outcomes: \( R_1R_1, R_1R_2, R_1B, R_1W, \)
\( R_2R_1, R_2R_2, R_2B, R_2W, \)
\( BR_1, BR_2, BB, BW, \)
\( WR_1, WR_2, WB, WW \)

Total number of possible outcomes = 16

Favourable outcomes: \( R_1R_1, R_1R_2, R_2R_1, R_2R_2 \)

Number of favourable outcomes = 4

\[ P(\text{both balls are red}) = \frac{4}{16} = \frac{1}{4} \]